

Resoluções

Capítulo 11

Transformações trigonométricas – Adição e subtração de arcos



ATIVIDADES PARA SALA

01 a) $\sin(45^\circ + 30^\circ) = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$

b) $\cos(120^\circ + 30^\circ) = \left(-\frac{1}{2}\right) \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = -\frac{\sqrt{3}}{2}$

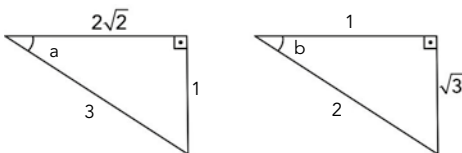
c) $\operatorname{tg}(180^\circ + 30^\circ) = \frac{0 + \frac{\sqrt{3}}{3}}{1 - 0 \cdot \frac{\sqrt{3}}{3}} = \frac{\sqrt{3}}{3}$

d) $\sin(45^\circ - 30^\circ) = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$

e) $\cos(30^\circ + 45^\circ) = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$

f) $\operatorname{tg}(45^\circ - 30^\circ) = \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}} = 2 - \sqrt{3}$

02



a) $\sin(a + b) = \sin a \cdot \cos b + \sin b \cdot \cos a = \frac{1}{3} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{2\sqrt{2}}{3} = \frac{2\sqrt{6} + 1}{6}$

b) $\cos(a + b) = \cos a \cdot \cos b - \sin a \cdot \sin b = \frac{2\sqrt{2}}{3} \cdot \frac{1}{2} - \frac{1}{3} \cdot \frac{\sqrt{3}}{2} = \frac{2\sqrt{2} - \sqrt{3}}{6}$

c) $\operatorname{tg}(a + b) = \frac{\operatorname{tg} a + \operatorname{tg} b}{1 - \operatorname{tg} a \cdot \operatorname{tg} b} = \frac{\frac{1}{2\sqrt{2}} + \sqrt{3}}{1 - \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{3}}{1}} = \frac{\frac{1 + 2\sqrt{6}}{2\sqrt{2}}}{\frac{2\sqrt{2} - \sqrt{3}}{2\sqrt{2}}} = \frac{1 + 2\sqrt{6}}{2\sqrt{2} - \sqrt{3}} \cdot \frac{2\sqrt{2} + \sqrt{3}}{2\sqrt{2} + \sqrt{3}} = \frac{2\sqrt{2} + \sqrt{3} + 8\sqrt{3} + 6\sqrt{2}}{8 - 3} = \frac{8\sqrt{2} + 9\sqrt{3}}{5}$

03 $\sin 20^\circ = 0,34 \rightarrow \cos 20^\circ = 0,94 \rightarrow \operatorname{tg} 20^\circ = 0,36$
 $\cos 30^\circ = 0,86 \rightarrow \sin 30^\circ = 0,5 \rightarrow \operatorname{tg} 30^\circ = 0,57$

a) $\sin(20^\circ + 20^\circ) = 0,34 \cdot 0,94 + 0,34 \cdot 0,94 \cong 0,63$

b) $\cos(20^\circ + 30^\circ) = 0,94 \cdot 0,86 - 0,34 \cdot 0,5 \cong 0,63$ ou
 $\cos 50^\circ = \sin 40^\circ = 0,63$

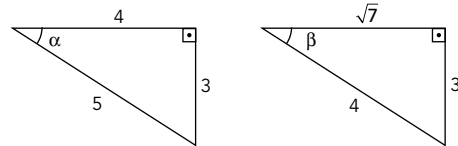
c) $\operatorname{tg}(20^\circ + 30^\circ) = \frac{0,36 + 0,57}{1 - 0,36 \cdot 0,57} \cong 1,17$

04 $\sin 165^\circ = \sin(120^\circ + 45^\circ) = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \left(-\frac{1}{2}\right) = \frac{\sqrt{6} - \sqrt{2}}{4}$

$\cos 165^\circ = \cos(120^\circ + 45^\circ) = \left(-\frac{1}{2}\right) \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{-\sqrt{2} - \sqrt{6}}{4}$

$\cos 165^\circ + \sin 165^\circ = -\frac{\sqrt{2}}{2}$

05



$\cos(\beta - \alpha) = \cos \beta \cdot \cos \alpha + \sin \beta \cdot \sin \alpha =$

$\left(-\frac{\sqrt{7}}{4}\right) \cdot \left(-\frac{4}{5}\right) + \frac{3}{4} \cdot \frac{3}{5} = \frac{4\sqrt{7} + 9}{20} = 4\sqrt{7} + 9$

ATIVIDADES PROPOSTAS

01) $\text{sen } x = \frac{4}{5} \Rightarrow \text{cos } x = \frac{3}{5}$

a) $\text{cos } y = \text{cos } (60^\circ - x) = \frac{1}{2} \cdot \frac{3}{5} + \frac{\sqrt{3}}{2} \cdot \frac{4}{5} = \frac{3+4\sqrt{3}}{10}$

b) $\text{tg } x = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$

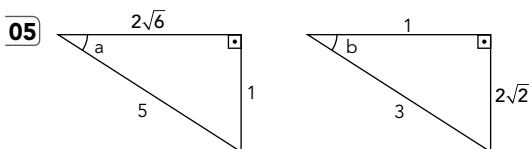
02) a) $\frac{\text{cos } (a-b) - \text{cos } (a+b)}{\text{sen } (a+b) + \text{sen } (a-b)} =$
 $= \frac{\text{cos } a \cdot \text{cos } b + \text{sen } a \cdot \text{sen } b - \text{cos } a \cdot \text{cos } b + \text{sen } a \cdot \text{sen } b}{\text{sen } a \cdot \text{cos } b + \text{sen } b \cdot \text{cos } a + \text{sen } a \cdot \text{cos } b - \text{sen } b \cdot \text{cos } a}$
 $= \frac{2\text{sen } a \cdot \text{sen } b}{2\text{sen } a \cdot \text{cos } b} = \text{tg } b \quad (\text{c.q.d.})$

b) $\text{sen } (a+b) \cdot \text{sen } (a-b) =$
 $= (\text{sen } a \cdot \text{cos } b + \text{sen } b \cdot \text{cos } a)(\text{sen } a \cdot \text{cos } b - \text{sen } b \cdot \text{cos } a) =$
 $= \text{sen}^2 a \cdot \text{cos}^2 b - \text{sen}^2 b \cdot \text{cos}^2 a =$
 $= \text{sen}^2 a \cdot (1 - \text{sen}^2 b) - \text{sen}^2 b \cdot (1 - \text{sen}^2 a) =$
 $= \text{sen}^2 a - \text{sen}^2 a \cdot \text{sen}^2 b - \text{sen}^2 b + \text{sen}^2 a \cdot \text{sen}^2 b =$
 $= \text{sen}^2 a - \text{sen}^2 b \quad (\text{c.q.d.})$

03) $\text{tg } (\gamma + \lambda) = \frac{\text{tg } \gamma + \text{tg } \lambda}{1 - \text{tg } \gamma \cdot \text{tg } \lambda} = \frac{\frac{1}{5} - \frac{1}{3}}{1 - \frac{1}{5} \cdot \left(-\frac{1}{3}\right)}$

$\frac{3-5}{15} = \left(-\frac{2}{15}\right) \cdot \frac{15}{16} = -\frac{1}{8}$
 $1 + \frac{1}{15}$

04) $33 = \frac{3 + \text{tg } y}{1 - 3\text{tg } y} \Rightarrow 3 + \text{tg } y = 33 - 99 \text{tg } y \Rightarrow$
 $100 \text{tg } y = 30 \Rightarrow \text{tg } y = \frac{3}{10}$



a) $\in 2^\circ$ quadrante b) $\in 3^\circ$ quadrante

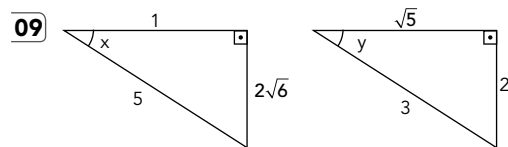
■ $\text{sen } (a+b) = \text{sen } a \cdot \text{cos } b + \text{sen } b \cdot \text{cos } a =$
 $= \frac{1}{5} \cdot \left(-\frac{1}{3}\right) - \frac{2\sqrt{2}}{3} \cdot \left(-\frac{2\sqrt{6}}{5}\right) = \frac{-1+8\sqrt{3}}{15} = -\frac{8\sqrt{3}-1}{15}$

■ $\text{cos } (a+b) = \text{cos } a \cdot \text{cos } b - \text{sen } a \cdot \text{sen } b =$
 $= \left(-\frac{2\sqrt{6}}{5}\right) \cdot \left(-\frac{1}{3}\right) - \frac{1}{5} \cdot \frac{-2\sqrt{2}}{3} = \frac{+2\sqrt{6} + 2\sqrt{2}}{15} = \frac{2\sqrt{2} \cdot (1 + \sqrt{3})}{15}$

06) $\frac{\text{tg } x + \text{tg } \frac{\pi}{4}}{1 - \text{tg } x \cdot \text{tg } \frac{\pi}{4}} = \frac{\frac{\sqrt{3}}{5} + 1}{1 - \frac{\sqrt{3}}{5}} = \frac{\sqrt{3} + 5}{5 - \sqrt{3}} \cdot \frac{5 + \sqrt{3}}{5 + \sqrt{3}} =$
 $= \frac{25 + 2 \cdot 5 \cdot \sqrt{3} + 3}{25 - 3} = \frac{28 + 10\sqrt{3}}{22} = \frac{14 + 5\sqrt{3}}{11}$

07) $\frac{\text{tg } x - \text{tg } y}{1 + \text{tg } x \cdot \text{tg } y} = -\frac{1}{3} \Rightarrow \frac{\text{tg } y}{1 + 2\text{tg}^2 y} = -\frac{1}{3}$
 $-3\text{tg } y = 1 + 2\text{tg}^2 y \Rightarrow 2\text{tg}^2 y + 3\text{tg } y + 1 = 0$
 $\text{tg } y = -1$ e $\text{tg } x = -2$ ou $\text{tg } y = -\frac{1}{2}$ e $\text{tg } x = -1$

08) $\text{sen} \left(x - \frac{\pi}{4}\right) + \text{cos} \left(x - \frac{\pi}{4}\right) = \frac{\sqrt{6}}{2}$
 $\text{sen } x \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \text{cos } x + \text{cos } x \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \text{sen } x = \frac{\sqrt{6}}{2}$
 $\sqrt{2} \cdot \text{sen } x = \frac{\sqrt{6}}{2} \Rightarrow \text{sen } x = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{3}$ ou $x = \frac{2\pi}{3}$
 $S = \left\{ \frac{\pi}{3}, \frac{2\pi}{3} \right\}$

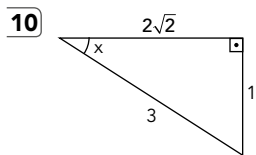


x) $\in 4^\circ$ quadrante y) $\in 3^\circ$ quadrante

■ $\text{sen } (y-x) = \text{sen } y \cdot \text{cos } x - \text{sen } x \cdot \text{cos } y =$
 $\left(-\frac{2}{3}\right) \cdot \frac{1}{5} - \left(-\frac{2\sqrt{6}}{5}\right) \cdot \left(-\frac{\sqrt{5}}{3}\right) = \frac{-2 - 2\sqrt{30}}{15} = -\frac{2(\sqrt{30} + 1)}{15}$

■ $\text{cos } (y-x) = \text{cos } y \cdot \text{cos } x + \text{sen } y \cdot \text{sen } x =$
 $\left(-\frac{\sqrt{5}}{3}\right) \cdot \frac{1}{5} + \left(-\frac{2}{3}\right) \cdot \left(-\frac{2\sqrt{6}}{5}\right) = \frac{4\sqrt{6} - \sqrt{5}}{15}$

■ $\text{tg } (y-x) = \frac{\text{tg } y - \text{tg } x}{1 + \text{tg } y \cdot \text{tg } x} \Rightarrow \text{tg } (y-x) = \frac{\frac{2\sqrt{5}}{5} + 2\sqrt{6}}{1 + \frac{2\sqrt{5}}{5} \cdot (-2\sqrt{6})} \Rightarrow$
 $\Rightarrow \text{tg } (y-x) = \frac{\frac{2\sqrt{5} + 10\sqrt{6}}{5}}{\frac{5 - 4\sqrt{30}}{5}} = \frac{2\sqrt{5} + 10\sqrt{6}}{5 - 4\sqrt{30}} \Rightarrow$
 $\Rightarrow \frac{2\sqrt{5} + 10\sqrt{6}}{5 - 4\sqrt{30}} \Rightarrow \text{tg } (y-x) = \frac{-(50\sqrt{5} + 18\sqrt{6})}{91}$



$x \in 1^{\text{a}}$ quadrante

$$\begin{aligned}\cos\left(\frac{\pi}{3} - x\right) &\Rightarrow \\ \Rightarrow \frac{1}{2} \cdot \frac{2\sqrt{2}}{3} + \frac{1}{3} \cdot \frac{\sqrt{3}}{2} &\Rightarrow \\ \Rightarrow \frac{2\sqrt{2} + \sqrt{3}}{6} &\Rightarrow\end{aligned}$$