Resoluções

Capítulo 9

Relações fundamentais e derivadas

ATIVIDADES PARA SALA

01 $\text{sen}^2\alpha + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 - \frac{49}{81} \Rightarrow$

$\Rightarrow \cos^2\alpha = \frac{32}{81} \Rightarrow \cos\alpha = \pm\frac{4\sqrt{2}}{9}$.

Como $\alpha \in 2^{\underline{o}}$ quadrante, $\cos\alpha = -\frac{4\sqrt{2}}{9}$.

02 $\text{sen}^2\gamma + \cos^2\gamma = 1 \Rightarrow \cos^2\gamma = 1 - \frac{1}{9}$

$\Rightarrow \cos^2\gamma = \frac{8}{9} \Rightarrow \cos\alpha = \pm\frac{2\sqrt{2}}{3}$

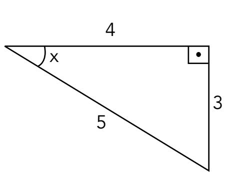
Como $\gamma \in 3^{\underline{o}}$ quadrante, tem-se:

$\text{sen}\,\gamma = -\frac{1}{3}, \cos\gamma = -\frac{2\sqrt{2}}{3}, \text{tg}\,\gamma = \frac{\sqrt{2}}{4},$

$\text{cossec}\,\gamma = -3, \sec\gamma = -\frac{3\sqrt{2}}{4}, \text{cotg}\,\gamma = 2\sqrt{2}.$

03 **B**

$\cos x = 0{,}8 = \frac{4}{5} \Rightarrow$



$\text{sen}\,x = \frac{3}{5} = 0{,}6$

$\text{tg}\,x = \frac{3}{4} = 0{,}75$

04 **D**

$\text{tg}\,x = \frac{\text{sen}\,x}{\cos x} \Rightarrow 1 = \frac{\text{sen}\,x}{\cos x} \Rightarrow \text{sen}\,x = \cos x$

Como $\text{sen}^2 x + \cos^2 x = 1$, tem-se:

$(\cos x)^2 + (\cos x)^2 = 1$

$2\cos^2 x = 1$

$\cos^2 x = \pm\frac{1}{2} \Rightarrow \cos x = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

Como **x** é um arco do $3^{\underline{o}}$ quadrante, o cosseno é negativo.

Assim, $\cos x = -\frac{\sqrt{2}}{2}$.

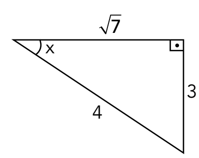
05 $\text{sen}^2 x + \cos^2 x = 1 \Rightarrow \frac{3y-1}{9} + \frac{y^2}{9} = 1$

$\Rightarrow y^2 + 3y - 10 = 0$

$\Rightarrow y = -5$ ou $y = 2$

ATIVIDADES PROPOSTAS

01 $\text{sen}\,x = -\frac{3}{4}$



x $\in 3^{\underline{o}}$ quadrante

$\cos x = -\frac{\sqrt{7}}{4}$ $\text{cossec}\,x = -\frac{4}{3}$

$\text{tg}\,x = \frac{3\sqrt{7}}{7}$ $\sec x = -\frac{4\sqrt{7}}{7}$

$\text{cotg}\,x = \frac{\sqrt{7}}{3}$

02 $\text{sen}^2\alpha + \cos^2\alpha = 1 \Rightarrow \frac{k-2}{k^2} + \frac{4}{k^2} = 1 \Rightarrow$

$k^2 - k - 2 = 0 \Rightarrow k = 2$ ou $k = -1$ (não convém)

Logo, k = 2.

03 a) $\text{sen}\,\alpha = \frac{1}{5} - \cos\alpha$

$\left(\frac{1}{5} - \cos\alpha\right)^2 + \cos^2\alpha = 1$

$\frac{1}{25} - \frac{2\cos\alpha}{5} + \cos^2\alpha + \cos^2\alpha = 1$

$50\cos^2\alpha - 10\cos\alpha - 24 = 0$

$25\cos^2\alpha - 5\cos\alpha - 12 = 0$

$\cos\alpha = \frac{4}{5}$ ou $\cos\alpha = -\frac{3}{5}$

$\text{sen}\,\alpha = -\frac{3}{5}$ ou $\text{sen}\,\alpha = \frac{4}{5}$

b) $\alpha \in$ 2º quadrante ou $\alpha \in$ 4º quadrante.

04 $\cos^4\alpha - \text{sen}^4\alpha = \underbrace{(\cos^2\alpha + \text{sen}^2\alpha)}_{1}(\cos^2\alpha - \text{sen}^2\alpha) =$

$= \cos^2\alpha - 1 + \cos^2\alpha = 2\cos^2\alpha - 1$ (c.q.d.)

05 $\text{sen}^2\alpha + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 - \frac{1}{9} \Rightarrow$

$\Rightarrow \cos^2\alpha = \frac{8}{9} \Rightarrow \cos\alpha = \pm\frac{2\sqrt{2}}{3}$

Como $\alpha \in$ 4º quadrante, tem-se:

$\cos\alpha = \frac{2\sqrt{2}}{3}$ e $\text{tg}\,\alpha = \frac{-\frac{1}{3}}{\frac{2\sqrt{2}}{3}} = -\frac{\sqrt{2}}{4}$

06 a) $\frac{\frac{\text{sen}^2 x}{\cos^2 x}}{1 + \frac{\text{sen}^2 x}{\cos^2 x}} = \frac{\text{sen}^2 x}{\cancel{\cos^2 x}} \cdot \frac{\cancel{\cos^2 x}}{\underbrace{\cos^2 x + \text{sen}^2 x}_{1}} = \text{sen}^2 x$

b) $\frac{1}{1 + \frac{\text{sen}^2 x}{\cos^2 x}} + \text{sen}^2 x = \frac{\cos^2 x}{\cos^2 x + \text{sen}^2 x} + \text{sen}^2 x = 1$

c) $\frac{\text{sen}^2 x + 2\,\text{sen}\,x\,\cos x + \cos^2 x}{\cos^2 x} - \frac{\text{sen}^2 x - 2\,\text{sen}\,x\,\cos x + \cos^2 x}{\cos^2 x}$

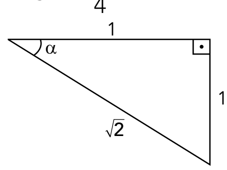
$\frac{\cancel{1} + 2\,\text{sen}\,x\,\cos x + 2\,\text{sen}\,x\,\cos x - \cancel{1}}{\cos^2 x} = \frac{4\,\text{sen}\,x\,\cos x}{\cos^2 x} = 4\,\text{tg}\,x$

d) $(\text{sen}^2 x - 1)^2 \cdot \left(1 + \frac{\text{sen}^2 x}{\cos^2 x}\right) = \frac{(\text{sen}^2 x - 1)^2}{\cos^2 x} =$

$\left(\frac{\text{sen}^2 x - 1}{\cos x}\right)^2 = \left(\frac{-\cos^2 x}{\cos x}\right)^2 = \cos^2 x$

07 $4\text{tg}^2\alpha + \text{tg}\alpha - 3 = 0 \therefore \text{tg}\,\alpha = -1$ ou $\text{tg}\,\alpha = \frac{3}{4}$ (não convém)

$\text{sen}\,\alpha = \frac{\sqrt{2}}{2}$ e $\cos\alpha = -\frac{\sqrt{2}}{2}$



08 **A**

$\text{tg}^2\alpha = \cos^2\alpha - \text{sen}^2\alpha$

$\text{tg}^2\alpha = 2\cos^2\alpha - 1$

$1 + \text{tg}^2\alpha = 2\cos^2\alpha$

$\sec^2\alpha = 2 \cdot \frac{1}{\sec^2\alpha}$

$(\sec^2\alpha)^2 = 2 \Rightarrow \sec^2\alpha = \sqrt{2}$

Como $1 + \text{tg}^2\alpha = \sec^2\alpha$, tem-se:

$1 + \text{tg}^2\alpha = \sqrt{2} \Rightarrow \text{tg}^2\alpha = \sqrt{2} - 1$

09 $\text{sen}^2\alpha + \cos^2\alpha = 1 \Rightarrow \frac{m^2}{25} + \frac{m^2 + 2m + 1}{25} = 1 \Rightarrow$

$2m^2 + 2m - 24 = 0 \Rightarrow m^2 + m - 12 = 0$

$m = 3$ ou $m = -4$

Soma $= 3 + (-4) = -1$.

10 **C**

$\frac{\cancel{(\text{sen}\,x + \cos x)}(\text{sen}^2 x - \text{sen}\,x\,\cos x + \cos^2 x)}{\cancel{\text{sen}\,x + \cos x}} = 1 - \text{sen}\,x \cdot \cos x$