Resoluções

Capítulo 20

Identidades trigonométricas

ATIVIDADES PARA SALA

01 a) $\text{sen}^4 x - \cos^4 x = (1 - \cos^2 x)^2 - \cos^4 x = 1 - 2\cos^2 x$

b) $(\sec x - \text{tg}\, x)\cdot(\sec x + \text{tg}\, x) \Rightarrow$

$\Rightarrow (\sec x)^2 - (\text{tg}\, x)^2 = (1 + \text{tg}^2 x) - \text{tg}^2 x = 1$

c) $1 - \text{sen}^2 x - \text{sen}^2 x = 1 - 2\,\text{sen}^2 x$

d) $1 - \cos^2 x - \cos^2 x = 1 - 2\cos^2 x$

e) $\left(\frac{\text{sen}\, x}{\cos x}\right)\cdot(\text{sen}\, x) + \frac{\cos^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$

02 a) $\text{tg}^2 x - 2\cdot \cancel{\text{tg}\, x}\cdot \cancel{\text{cotg}\, x} + \text{cotg}^2 x = \text{tg}^2 x + \text{cotg}^2 x - 2$

b) $\text{tg}^2 x + 2\cdot \cancel{\text{tg}\, x}\cdot \cancel{\text{cotg}\, x} + \text{cotg}^2 x = \text{cotg}^2 x + \text{tg}^2 x + 2$

03 $\dfrac{\frac{1}{\cos x} - \frac{1}{\text{sen}\, x}}{\frac{1}{\cos x} + \frac{1}{\text{sen}\, x}} = \dfrac{\frac{\text{sen}\, x - \cos x}{\cancel{\text{sen}\, x}\cos x}}{\frac{\text{sen}\, x + \cos x}{\cancel{\text{sen}\, x}\cos x}} = \dfrac{\text{tg}\, x - 1}{\text{tg}\, x + 1}$

04 $\dfrac{\text{sen}\, x}{\cos x} - \dfrac{\cos x}{\text{sen}\, x} = \dfrac{\text{sen}^2 x - \cos^2 x}{\text{sen}\, x\cdot\cos x} = \dfrac{1 - 2\cos^2 x}{\text{sen}\, x\cdot\cos x}$

05 $\text{sen}^2 x - \text{sen}^2 x\cdot\cos^2 x = \text{sen}^2 x\cdot(1 - \cos^2 x) = \text{sen}^2 x\cdot\text{sen}^2 x = \text{sen}^4 x$

ATIVIDADES PROPOSTAS

01 $a^2\cdot\text{sen}^2 x - \cancel{2ab\cdot\text{sen}x\cdot\cos x} + b^2\cdot\cos^2 x + a^2\cdot\cos^2 x +$

$+ \cancel{2ab\cdot\text{sen}x\cdot\cos x} + b^2\cdot\text{sen}^2 x =$

$a^2\cdot(\text{sen}^2 x + \cos^2 x) + b^2\cdot(\cos^2 x + \text{sen}^2 x) = a^2 + b^2$

02 $\dfrac{\frac{\cos^2 x}{\text{sen}^2 x}}{1 + \frac{\cos^2 x}{\text{sen}^2 x}} = \dfrac{\cos^2 x}{\text{sen}^2 x + \cos^2 x} = \cos^2 x$

03 $\dfrac{1}{\cos^2 x} + \dfrac{1}{\text{sen}^2 x} = \dfrac{\text{sen}^2 x + \cos^2 x}{\cos^2 x\cdot\text{sen}^2 x} =$

$= \dfrac{1}{\cos^2 x\cdot\text{sen}^2 x} = \sec^2 x\cdot\text{cossec}^2 x$

04 $\cos^2 a - \cos^2 b + \text{sen}^2 a - \text{sen}^2 b = 1 - 1 = 0$

05 $\begin{cases}\text{sen}^2 x + \cos^2 x = 1\\ \text{sen}^2 y + \cos^2 y = 1\end{cases} \Rightarrow \text{sen}^2 x + \cos^2 x = \text{sen}^2 y + \cos^2 y \Rightarrow$

$\text{sen}^2 x - \text{sen}^2 y = \cos^2 y - \cos^2 x \Rightarrow$

$(\text{sen}\, x + \text{sen}\, y)\cdot(\text{sen}\, x - \text{sen}\, y) = (\cos y + \cos x)\cdot(\cos y - \cos x) \Rightarrow$

$\dfrac{\text{sen}\, x + \text{sen}\, y}{\cos y - \cos x} = \dfrac{\cos y + \cos x}{\text{sen}\, x - \text{sen}\, y}$

06 $1 - \text{tg}^2 x = 1 - (\sec^2 x - 1) = 2 - \sec^2 x$

07 $\dfrac{\cos x + \frac{\cos x}{\text{sen}\, x}}{\frac{\text{sen}\, x}{\cos x} + \frac{1}{\cos x}} = \dfrac{\frac{\text{sen}\, x\cdot\cos x + \cos x}{\text{sen}\, x}}{\frac{\text{sen}\, x + 1}{\cos x}} =$

$\dfrac{\cos x\cdot\cancel{(1 + \text{sen}\, x)}}{\text{sen}\, x}\cdot\dfrac{\cos x}{\cancel{1 + \text{sen}\, x}} = \cos x\cdot\text{cotg}\, x$

08 $\dfrac{\text{sen}^2 x - \cos^2 y + \cos^2 x\cdot\cos^2 y}{\cos^2 x\cdot\cos^2 y} =$

$\dfrac{\text{sen}^2 x - (1 - \text{sen}^2 y) + (1 - \text{sen}^2 x)\cdot(1 - \text{sen}^2 y)}{\cos^2 x\cdot\cos^2 y} =$

$\dfrac{\cancel{\text{sen}^2 x} - \cancel{1} + \cancel{\text{sen}^2 y} + \cancel{1} - \cancel{\text{sen}^2 y} - \cancel{\text{sen}^2 x} + \text{sen}^2 x\cdot\text{sen}^2 y}{\cos^2 x\cdot\cos^2 y} =$

$\dfrac{\text{sen}^2 x\cdot\text{sen}^2 y}{\cos^2 x\cdot\cos^2 y} = \text{tg}^2 x\cdot\text{tg}^2 y$

09 $\left(\dfrac{1}{\text{sen}\, x} - \dfrac{\cos x}{\text{sen}\, x}\right)^2 = \dfrac{(1 - \cos x)^2}{\text{sen}^2 x} =$

$\dfrac{(1 - \cos x)^2}{(1 - \cos x)\cdot(1 + \cos x)} = \dfrac{1 - \cos x}{1 + \cos x}$

10 $\sec^2 x = 1 + \text{tg}^2 x \Rightarrow \sec^2 x - \text{tg}^2 x = 1 \Rightarrow$

$(\sec x + \text{tg}\, x)\cdot(\sec x - \text{tg}\, x) = 1 \Rightarrow \sec x - \text{tg}\, x = \dfrac{1}{\sec x + \text{tg}\, x}$