

# Resoluções

## Capítulo 20

### Logaritmos – Definição; Consequências da definição



#### ATIVIDADES PARA SALA

**01** a)  $10 = 0,01^x \Rightarrow 10 = 10^{-2x} \Rightarrow x = -\frac{1}{2}$

b)  $\sqrt[3]{2^4} = 8^x \Rightarrow 2^{\frac{4}{3}} = 2^{3x} \Rightarrow x = \frac{4}{9}$

c)  $\sqrt[4]{8} = \left(\frac{1}{4}\right)^x \Rightarrow 2^{\frac{3}{4}} = 2^{-2x} \Rightarrow x = -\frac{3}{8}$

d)  $0,25 = (\sqrt[3]{2})^x \Rightarrow 2^{-2} = 2^{\frac{x}{3}} \Rightarrow x = -6$

e)  $\frac{4}{9} = 1,5^x \Rightarrow \left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right)^{-x} \Rightarrow x = -2$

**02** a)  $\log_{2^{-3}} 2^{\frac{3}{4}} = \frac{\cancel{x}}{4} \cdot \left(-\frac{1}{\cancel{x}}\right) \cdot 1 = -\frac{1}{4}$

b)  $\log_{10^2} 10^{\frac{1}{3}} = \frac{1}{3} \cdot \frac{1}{2} \cdot 1 = \frac{1}{6}$

c)  $\log_3 3^{-4} = -\frac{4}{3}$

d)  $\log_{3^{-1}} 3^{\frac{2}{3}} = -\frac{2}{3}$

e)  $\log_{3^{\frac{3}{2}}} 3^{\frac{3}{2}} = \frac{3}{2} \cdot (-2) = -3$

**03 E**

$\log_{2\sqrt{3}} 144 = x$

$(2\sqrt{3})^x = 144$

$(\sqrt{4 \cdot 3})^x = 12^2$

$(\sqrt{12})^x = 12^2$

$12^{\frac{x}{2}} = 12^2 \Rightarrow \frac{x}{2} = 2$

$x = 4$

**04** a)  $\log_{\sqrt[3]{100}} \sqrt{0,1} = x \Rightarrow \left(\frac{1}{10}\right)^{\frac{1}{3}} = 100^{\frac{x}{3}} \Rightarrow 10^{-\frac{1}{3}} = 10^{\frac{2x}{3}} \Rightarrow x = -\frac{3}{4}$

$\log_{\sqrt[3]{0,5}} \sqrt{8} = y \Rightarrow \sqrt{8} = \sqrt[3]{0,5^y} \Rightarrow 2^{\frac{3}{2}} = 2^{-\frac{y}{3}} \Rightarrow y = -\frac{9}{2}$

$x + y = -\frac{3}{4} - \frac{9}{2} = \frac{-3-18}{4} = -\frac{21}{4}$

b) 
$$\begin{cases} \log_{0,1} 0,01 = x \Rightarrow 10^{-2} = 10^{-x} \Rightarrow x = 2 \\ \log_{\sqrt{2}} 0,25 = y \Rightarrow 2^{-2} = 2^{\frac{y}{2}} \Rightarrow y = -4 \\ \log_{25} 0,008 = z \Rightarrow 5^{-3} = 5^{2z} \Rightarrow z = -\frac{3}{2} \end{cases}$$
  

$$\Rightarrow 2 - 3 \cdot (-4) + \frac{1}{2} \cdot \left(-\frac{3}{2}\right) = \frac{53}{4}$$

**05**  $0 < x \neq 1$   
 $x^{-2} = 9$

$\frac{1}{x^2} = 9 \Rightarrow 9x^2 = 1 \Rightarrow x^2 = \frac{1}{9} \Rightarrow x = \frac{1}{3}$



#### ATIVIDADES PROPOSTAS

**01** a)  $\log_2 2^{-1} = -1$

d)  $\log_{2^{-1}} 2^5 = -5$

b)  $\log_4 1 = 0$

e)  $\log_{\left(\frac{5}{2}\right)^{-2}} \left(\frac{5}{2}\right)^4 = \cancel{4}^2 \cdot \left(-\frac{1}{\cancel{2}}\right) = -2$

c)  $\log_{\frac{1}{5}} \frac{1}{5} = 1$

**02** a)  $x = \sqrt[3]{5^3} = 5$

b)  $x = 10^{-3} = \frac{1}{1000}$

c)  $x + 1 = 8 \Rightarrow x = 7$

d)  $10 = x^2 \Rightarrow x = \sqrt{10}$

e)  $16 = 8x^3 \Rightarrow x = \sqrt[3]{2}$

f)  $5 = x - 1 \Rightarrow x = 6$

**03** a)  $\log_2 (\log_3 3^4) = \log_2 4 = 2$

b)  $\log_5 [\log_3 (\log_4 4^3)] = \log_5 (\log_3 3) = \log_5 1 = 0$

**04** Considerando  $\log_5 125 = x$ , tem-se:

$5^x = 125 \Rightarrow 5^x = 5^3 \Rightarrow x = 3$

Assim:

$\log_{\frac{1}{3}} 3 = y \Rightarrow \left(\frac{1}{3}\right)^y = 3 \Rightarrow (3^{-1})^y = 3^1 \Rightarrow -y = 1 \Rightarrow y = -1$

05 a)  $2x - 1 > 0 \Rightarrow x > \frac{1}{2}$

b)  $\left. \begin{array}{l} x - 1 > 0 \Rightarrow x > 1 \\ 1 \neq x - 2 > 0 \Rightarrow 3 \neq x > 2 \end{array} \right\} \text{Interseção} \Rightarrow 3 \neq x > 2$

c)  $\left. \begin{array}{l} -2x + 6 > 0 \Rightarrow x < 3 \\ 1 \neq x > 0 \end{array} \right\} \text{Interseção} \Rightarrow 0 < x < 1 \text{ ou } 1 < x < 3$

06  $x^2 - 5x + 6 > 0 \Rightarrow x < 2 \text{ ou } x > 3 \Rightarrow 1 \neq x > 0$   
 Interseção  $\Rightarrow D = \{x \in \mathbb{R} \mid 0 < x < 1 \text{ ou } 1 < x < 2 \text{ ou } x > 3\}$

07  $\log_{y^2} x^3 = \frac{3}{2} \log_y x = \frac{3}{2} \cdot 3 = \frac{9}{2}$

08  $(8\sqrt{2})^x = 0,25$   
 $\left(2^3 \cdot 2^{\frac{1}{2}}\right)^x = \frac{25}{100} \Rightarrow \left(2^{3+\frac{1}{2}}\right)^x = \frac{1}{4} \Rightarrow \left(2^{\frac{7}{2}}\right)^x = 4^{-1} \Rightarrow$   
 $2^{\frac{7x}{2}} = (2^2)^{-1} \Rightarrow 2^{\frac{7x}{2}} = 2^{-2} \Rightarrow \frac{7x}{2} = -2 \Rightarrow 7x = -4 \Rightarrow$   
 $x = -\frac{4}{7}$

09 Sendo  $9 = 3^2$ , tem-se:

$$(3^2)^{\log_3 7} = (3^{\log_3 7})^2 = 7^2 = 49$$

10  $\log a = \frac{1}{2} \Rightarrow 10^{\frac{1}{2}} = a \Rightarrow a = \sqrt{10}$

$$\log b = -1 \Rightarrow 10^{-1} = b \Rightarrow b = \frac{1}{10}$$

$$\log_{\frac{1}{10}} \sqrt{10} = -\frac{1}{2}$$